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How scale-free networks and large-scale collective cooperation emerge in complex homogeneous social systems

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We study how heterogeneous degree distributions and large-scale collective cooperation in social networks emerge in complex homogeneous systems by a simple local rule: learning from the best in both strategy selections and linking choices. The prisoner's dilemma game is used as the local dynamics. We show that the social structure may evolve into single-scale, broad-scale, and scale-free (SF) degree distributions for different control parameters. In particular, in a relatively strong-selfish parameter region the SF property can be self-organized in social networks by dynamic evolutions and these SF structures help the whole node community to reach a high level of cooperation under the poor condition of a high selfish intention of individuals.

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Scale-free (SF) network structures, which have been observed widely in social systems, have attracted great interest in the past few years [1,2]. So far, SF networks have been theoretically constructed only in various growing and attachment models [2–5] under a few artificial linking rules. Up to date, an important problem has not yet been touched: *how these interesting social structures can be developed from node dynamics based on some basic abilities and instincts of social members*. For instance, learning from the best (LFB) is one of the primary abilities of social individuals [6]. For each individual, learning is based on limited information among few familiar neighbors. Moreover, a basic characteristic of a social member is selfishness, because the short-term anticipation of selfishness can achieve high benefits; e.g., this is the case in the simple two-player games of the prisoner's dilemma (PD) [6–11]. Recently, some works have revealed that certain hub structures and degree heterogeneities, including broad-scale degree distributions, can emerge from homogeneous PD networks through coevolution of strategies and structures [12–15]. It remains interesting to ask whether the SF structures may emerge from the local PD and LFB interactions and how these complex structures can influence the global collective social cooperations if the answer to the first question is yes.

In this paper, we consider social networks with fixed local links and adjustable long-range links (LRLs) which are assumed to be practical in realistic social systems. Nodes in the network play PD games with other nodes in connections, following the rule of LFB to adjust their strategies and LRLs. A significant phenomenon observed is that social structures with single-scale, broad-scale, and SF distributions can be constructed in the same topological model for different control parameters. The most interesting result is that in a relatively strong-selfish parameter region SF social structures can be self-organized from the "first social principles"—i.e., from the basic individual abilities of local PD and LFB dynamics. We show that SF structures can be constructed through dynamic coevolution rather than through network growing. Moreover, these SF structures have the function of helping society to reach high level of cooperation under the hard condition of a strong local selfish intention of social members.

Specifically, the network has the following characteristics.

(i) *Local and long-range link networks* (LLN). All nodes, representing individuals in the game, are distributed in a two-dimensional (2D) lattice. All individuals have nearest-neighbor interactions shown in Fig. 1 by n local lines without arrows ($n=4$, 3, and 2 for internal, edge, and corner nodes, respectively). These local interactions are determined by the spatial neighborhood, and they are reasonably assumed to be fixed during the whole process of the game. On the other hand, all individuals have limited capacities to establish LRLs. For simplicity, we assume that each node can actively take a single LRL. All LRLs are directed by arrows. For a given node (e.g., node A in Fig. 1) a link is called active if the arrow of the LRL is from A while passive if the arrow is toward A. A node having an active LRL aiming at itself or at one of its local neighbors is prohibited. It is obvious that each node must have one active link and the number of passive links can vary.

(ii) *Dynamic PD game*. The dynamics of nodes is governed by the two-player PD game. In its standard form, each player may choose either to cooperate, C, or to defect, D, in

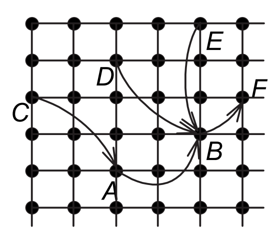

FIG. 1. Schematic structure of a LLN system. Each node connects with n local neighbor nodes in 2D network with fixed links and has one long-range active link to a node randomly chosen (arrow from a given node to a target node). For example, link AB is an active link of node A and a passive link of node B. And in the figure node B has one long-range active link, three passive links, and four fixed local links.

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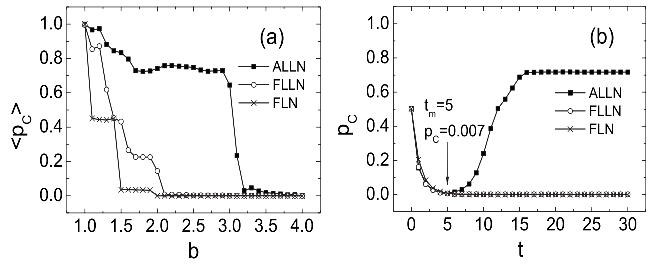

FIG. 2. Behaviors and evolutions of cooperations for different network structures. β=0.1 for this and all the following figures. (a) Average percentages of cooperators in the stationary states of various network systems as the functions of the payoff parameter b (averages over 10^3 random initial strategies with p_C=0.5 and random long-range links). (b) p_C's evolve with t for various systems at b=2.7. System size of 100×100 is taken for (a) and (b).

any one encounter. If both players choose C, both get a payoff of R; if one defects while the other cooperates, D gets T, while C gets S; if both defect, both get P, where $T>R>P>S$ [7,16]. Here we adopt the values $R=1$, $T=b>1$, $S=P=0$ [9,17,18]. Parameter b characterizes the temptation to defection against cooperation—i.e., the relative selfish intention of individuals. Each node plays the classical PD games with all nodes connected. The total payoff of an individual is the sum of the payoffs obtained in his two-player games with all other connected nodes:

$$P_A = \sum_{i=1}^{N} P_{Ai}\delta_{Ai}. \tag{1}$$

$\delta_{Ai}=1$ when A has an interaction with i, else 0.

(iii) *Learning from the best*. Individuals are capable of adjusting their strategies and rewiring their active LRLs according to their reachable information in the previous round of the game. Here we use a simple law of adjustments: learning from the best. Each individual (say, node A) compares the payoff of himself with those of all his connected neighbors in the previous round and finds out the individual with the best payoff (or choose one with equal probabilities if there are multiple best neighbors). If node A has the largest payoff, A keeps its own previous strategy and LRL; otherwise, A learns from the best neighbor (say, node B) by simultaneously taking the same strategy of the best one and rewiring its active LRL to that of B (if the new LRL does not achieve more profit, keeps the old link) with probability

$$W(A\to B) = 1/(1+e^{-\beta(P_B-P_A)}), \tag{2}$$

where P_A and P_B denote the total payoffs of individuals A and B, respectively, and β characterizes the noise effects [19]. Therefore, we consider the dynamic coevolution of both game strategies and linking structure of individuals.

We first study the evolution behavior of cooperation of our system. For comparison we also consider some other networks with fixed linking structures. In Fig. 2(a) we plot the average functions of cooperators, p_C, for the stationary solutions of some network systems, where ALLN represents our networks with fixed local and adjustable LRLs; FLLN denotes networks with fixed local and fixed LRLs; FLN shows networks with fixed local links only. The same LFB law is applied in all these three kinds of systems for the strategy evolutions. It is observed that in a large range of b the cooperation level in the ALLN system is much higher than those of other systems with fixed network structures. In order to compare the actual dynamic evolution of strategies, we plot, in Fig. 2(b), p_C's against time t for different systems for relatively large b which favors defectors. In all systems of fixed networks, p_C's decrease monotonously to zero. p_C in our ALLN system decreases also for small t, in a manner similar to other systems [$p_C(\min)\approx0.007$]. However, it is surprising to observe that after a certain time, p_C starts to increase as time and finally saturates to a rather high level in an asymptotic stationary state. This interesting feature is due to the self-organization of long-range connections by LFB as we will explain later.

Now we go further to study the evolution of network structure of the ALLN system. In Figs. 3(a)–3(c) we plot the cumulative degree distributions of passive LRLs of nodes, $P(k)$, for different b values with k being the number of passive links of a given node. We find essentially different scaling features of $P(k)$ distributions. In (a) for small b the probability has an exponential (single-scale) distribution, showing certain random connections between the nodes. In (b), $P(k)$ shows a power law $P(k)\propto k^{-\gamma}$ for small k and an exponential decay tail for relatively large k—i.e., broad-scale distribution. The most interesting feature is observed in Fig. 3(c) where SF distribution $P(k)\propto k^{-\gamma}$, $\gamma\approx1$, is observed practically for the whole range of k. We find SF networks realized as attracting structures of coevolution of node dynamics, and this is essentially different from all previously known SF networks constructed by various preferential attachment rules. The cumulative degree distribution $P(k)$ changes continuously from single scale to broad scale to SF scalings by increasing b and finally to states with full defectors, and no critical transitions between these stages are observed. Roughly speaking, one may observe single-scale scaling for $1\lesssim b\lesssim1.9$ broad-scale scaling for $1.9\lesssim b\lesssim2.5$ and SF for $2.5\lesssim b\lesssim3.0$. For $b>3.0$ all nodes are conquered by defectors (note it is still possible that nonzero p_C can be observed in the range $b\gtrsim3.0$ for some less probable initial distributions).

In the two-player games the defective strategy prevails always by the PD rule unless cooperators form clusters and get benefits from the mutual and collective operations. For the case of weak selfishness, randomly formed small cooperator clusters [exponential decay of $P(k)$] are sufficient for the cooperator hubs to resist the invasion of defectors and to reach the cooperator-defector balance [Fig. 3(d)]. By increasing b (i.e., increasing the intensity of selfishness), clusters randomly formed with small passive connections are not strong enough to resist the defector invasions. The LFB rule is favorable for cooperator hubs with a large degree of links to grow and survive [Fig. 3(e)], and this produces the broad-scale distribution of Fig. 3(b). In particular, with SF network structure cooperation can prevail for rather large b [Fig. 3(f), b=2.7] at which defectors dominate all the fixed networks [Fig. 2(a)]. This desired feature is due to the self-organizations of cooperators by LFB: (i) Some cooperators

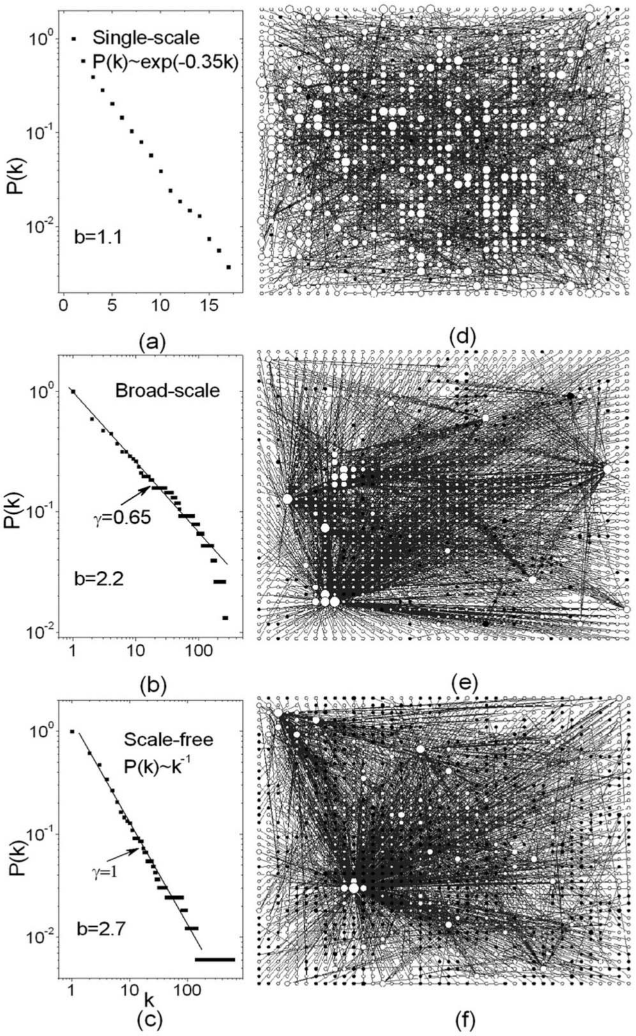

FIG. 3. Cumulative stationary degree distributions of passive links $P(k)$ [(a),(b),(c)] and asymptotic stationary patterns of ALLN systems for different parameter b's[(d),(e),(f)]. Arbitrary initial strategy distributions with $p_C=0.5$ are used. (a), (d) $b=1.1$; (b), (e) $b=2.2$; (c), (f) $b=2.7$. In (d), (e), and (f), open circles (○) represent cooperators, solid circles (●) denote defectors, and solid gray lines show LRLs. The sizes of the open and solid circles are proportional to the logarithms of the numbers of links of the given nodes in each figure, and different scales are used for different figures. The maximum hubs have links 17 in (a), 273 in (b), and 609 in (c). System size $N=40\times40$ is used in Figs. 3 and 4.

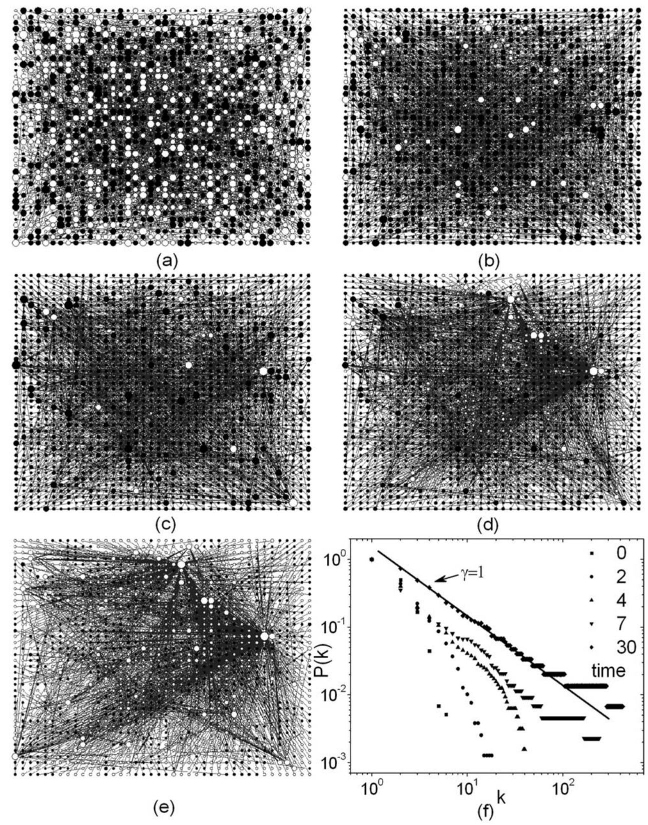

FIG. 4. Self-organization of scale-free network in a dynamic coevolution of individual strategies and network structure with PD and LFB rules. $b=2.7$. (a)–(e) The same as Figs. 3(d)–3(f) with pattern snapshots at different times plotted. (a) $t=0$. Initial random strategy and LRL distributions. (b) $t=2$, $p_C=0.052$. The number of defectors (cooperators) increases (decreases). (c) $t=t_m=4$, $p_C=0.0125$. The percentage of defectors arrives at the maximum. Small cooperator clusters remain. (d) $t=7$, $p_C=0.15$. Some defectors adopt cooperative strategy and cooperator clusters expand by LFB. (e) $t=30$. Stationary pattern with scale-free scaling and $p_C=0.66$ is approached. The maximum cooperator hubs have links (a) 6, (b) 18, (c) 40, (d) 241, and (e) 442. (f) Cumulative degree distributions of passive long-range links at time $t=0,2,4,7,30$.

form hubs by connecting passively with large numbers of other cooperators. In this way the hub cooperators can survive for large b against connected defectors by using the benefits obtained from other connected cooperators. (ii) Many cooperators with small connection degree can survive for large b also by linking with one of these hubs and learning from these hubs. (iii) Moreover, some defectors of low degree can also survive asymptotically by connecting with cooperator nodes without conquering them.

In order to understand the correspondence between the cooperation increase in Fig. 2(b) for $t>t_c$ and the formation of SF networks, we show the dynamic coevolution of individual strategies and network structure in Fig. 4. Initially defectors and cooperators distribute randomly in the network with equal probabilities $p_C=p_D=0.5$ and the LRLs are also set randomly [Fig. 4(a)]. At the early stage of evolution defectors and cooperators form small hubs with random connections and low degrees of single-scale links. In this stage the number of defectors increases quickly due to the fact that the two-player interactions are favorable strongly to the selfish individuals [Fig. 4(b)]. In the second stage, defectors almost dominate the system [Fig. 4(c)] because large-scale collective operations between cooperators have not yet been established then. In this stage we observe that the degrees of cooperator hubs increase, by attracting both cooperators and defectors with the LFB law. On the other hand, some cooperator hubs may change to defector hubs when some of their defector neighbors gain profits more than the hubs. Moreover, defective hubs are unstable because nodes linking with them prefer to rewire their links due to the fact that they cannot enjoy the best payoffs by connecting with these hubs.

Therefore, expansion of cooperator hubs, invasion of defective strategy into cooperator hubs, and collapses of defector hubs are the characteristic features of the second stage. A broad-scale degree distribution of connections emerges in this stage [Fig. 4(f)]. At t_m, we observe $p_C(\min) \approx 0.01$. However, this small cluster of cooperators turns out to be stable. Cooperators get satisfactory payoffs from mutual cooperations and keep their cooperation strategy by learning from the connected cooperator hubs. In the third stage ($t > t_m$) the surviving cooperator hubs expand due to the fact that more and more nodes link to cooperator hubs by learning from the neighbors with the best performance [Fig. 4(d)]. These reorganizations effectively enlarge the power-law scaling region of the broad-scale degree distribution. In this stage the number of cooperators increases rapidly because many defectors change their strategies with the LFB rule. Finally, the node connections are attracted to a stationary SF network structure [Fig. 4(e)] with almost the entire power-law degree distribution [Fig. 4(f) for $t=30$]. For systems with larger sizes we find SF properties similar to Figs. 3(c) and 4(f) for $t=30$ except that the k length of the power-law scaling is considerably enlarged.

In conclusion, we have studied the coevolution of strategy and linking structure in social networks by the PD and LFB interactions. Rich heterogeneities of social networks with single-scale, broad-scale, and SF scalings may result from coevolution in different PD conditions. In particular, our simple model suggests that the SF feature, which has been shown to be so pervasive in complex systems, can arise from dynamic evolution via a self-organizing mechanism through individual learning ability. And this SF structure enables networks to reach a high level of cooperation against defector invasions with strong selfish intention of individuals. These results shed light on understanding how complex networks with global collective cooperation can emerge from social individuals with local and primary abilities and instincts.

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